Electromagnetics and seismics: the deep connections

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Summary

Seismic waves obey the wave equation, and low-frequency EM waves obey the diffusion equation. Despite this, there is a deep similarity in their properties. Both wave types may be analyzed as a superposition of plane waves which are both dispersive and attenuative; the principal difference is that for seismics, these effects are weak, whereas for EM, they are strong. This means that *all* seismic processing algorithms which do *not* assume low dispersion and attenuation may be used to process properly acquired EM data; this is the majority of the seismic toolkit. In particular, EM data may be *imaged*, seismic-style, rather than mathematically inverted, EM style.

With both data-types, the signal from the subsurface is weak, so it is best to detect it without competition from a concurrent source. This means that controlled sources of EM should be *impulsive* sources ("ISEM") rather than *continuous* sources ("CSEM"). At low frequency, the EM phase velocity is a few km/s, comparable to seismic velocities. Hence, seismic-style acquisition, including the measurement of moveout, is feasible for EM data.

For seismics, the Quality factor $\mathbf{Q_{seis}}$ is a large material property, to be determined from the data; whereas $\mathbf{Q_{EM}}$= ½ is a property of the equation.

Introduction

Seismology is certainly the most effective single technology in exploration geophysics, yet there are many instances where seismic data and analysis are not adequate, by themselves, to answer important subsurface questions. In such instances, it is frequently true that *the rest of* geophysics can play an important role. *Electromagnetics* may be especially useful, as a complement to seismics, because, at low frequencies, the governing electromagnetic material property is electrical resistivity, which depends strongly upon hydrocarbon saturation (of course, this fact is the foundation of the logging industry).

The fundamental equations which govern electromagnetic propagation are different from those that govern seismic propagation, so different mathematical techniques have historically evolved to deal with the two types of data. However, there is also a deep *similarity* in the mathematics, which may be used to understand both technologies in a unified way (Ursin, 1983). This unified understanding serves to motivate a new prescription of how best to *acquire* and *process* electromagnetic surveys.

A Common Mathematical Basis

Starting first with elastic seismic theory, the simplest theoretical approach is through the 1-D vector wave equation for an isotropic homogeneous medium:

$$\frac{\partial^2 \vec{u}}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 \vec{u}}{\partial t^2} = 0 \qquad (1)$$

where $\vec{\boldsymbol{u}}(\boldsymbol{x},\boldsymbol{t})$ is the vector particle displacement (variable in 1-D space $\boldsymbol{x}$ and time $\boldsymbol{t}$) within the wave. The medium itself is characterized by the velocity $\boldsymbol{v}$. It is well-known to all geophysicists that all solutions to (1) may be represented as a sum of Fourier basis terms:

$$\vec{\boldsymbol{u}}(\boldsymbol{x},\omega) = \vec{\boldsymbol{u}}_0(\omega)\,\mathrm{e}^{i\omega(\boldsymbol{t}-\boldsymbol{x}/\boldsymbol{v})} \qquad (2)$$

with different angular frequencies $\boldsymbol{\omega}$. For eventual comparison to the electromagnetic case, it is best to specify this wave as a *shear* wave, with the displacement vector $\vec{\boldsymbol{u}}_0$ perpendicular to the propagation direction $\boldsymbol{x}$**.**

The anelastic generalization of (1) is not so well-known, but in simplest terms, it is the same as Eqn. (1), but with a complex velocity $\boldsymbol{v}$, resulting in solutions of the form (O'Connell and Budiansky, 1978):

$$\vec{\boldsymbol{u}}(\boldsymbol{x},\omega) = \vec{\boldsymbol{u}}_0(\omega)\,\mathrm{e}^{i\omega(\boldsymbol{t}-\boldsymbol{x}/\boldsymbol{v}_{phs})}\,\mathrm{e}^{-\omega \boldsymbol{x}/2\boldsymbol{v}_{phs}\boldsymbol{Q}_{seis}} \qquad (3)$$

The principal difference between Eqns. (3) and (2) is the appearance in Eqn. (3) of an exponential attenuation term, on the right. The degree of attenuation is controlled by the (real, dimensionless) "quality parameter" $\boldsymbol{Q_{seis}}$, which depends *in principle* upon frequency (Futterman, 1962), as does the (real) phase velocity $\boldsymbol{v_{phs}}$. In general, this attenuation/dispersion depends upon poorly understood physical mechanisms, and must be determined from the data itself. Normally, $\boldsymbol{Q_{seis}}$ is a large number (e.g. 20-100), so that often in exploration geophysics we make the approximation that $\boldsymbol{Q}_{seis} \approx \infty$, and (3) reduces to (2).

Consider next the corresponding electromagnetic equation (*c.f., e.g.* Jackson, 1962). Maxwell's (second-order) wave/diffusion equation for the electric field $\vec{\boldsymbol{E}}(\boldsymbol{x},\boldsymbol{t})$ (within a non-magnetic material) is:

$$\frac{\partial^2 \vec{\boldsymbol{E}}}{\partial \boldsymbol{x}^2} - \frac{\boldsymbol{n}^2}{\boldsymbol{c}^2}\frac{\partial^2 \vec{\boldsymbol{E}}}{\partial \boldsymbol{t}^2} - \frac{\mu_0}{\rho}\frac{\partial \vec{\boldsymbol{E}}}{\partial \boldsymbol{t}} = 0 \qquad (4)$$

where $\boldsymbol{c}$ is the speed of light in free space, $\boldsymbol{n}$ is the index of refraction (>1) of the medium, $\boldsymbol{\mu_0}$ = 4π × 10^{-7} N/A^2 is the

DOI http://dx.doi.org/10.1190/segam2014-0798.1

magnetic permeability of free space, and ρ is the electrical resistivity of the medium. The remarkable difference between Equations (4) and (1) is the presence of the third term on the left, with a *single* time-derivative.

It is well-known that all solutions to Equation (4) may be represented as a sum of Fourier basis terms, just as in the seismic case. In 1-D:

$$\vec{E}(\vec{x},\omega) = \vec{E}_0(\omega)\,e^{i(\omega t - kx)} = \vec{E}_0(\omega)\,e^{i\omega(t - x/v_{EM})} \quad (5)$$

where $k=\omega/v_{EM}$ is the wavenumber. The complex phase velocity v_{EM} is given by (Jackson (1962), Ursin (1983) :

$$v_{EM} = \frac{c}{n}\left(1 - i\,\frac{\mu_0}{\omega\rho}\left(\frac{c}{n}\right)^2\right)^{-1/2} \underset{low\,f}{\rightarrow} \sqrt{\frac{2\rho\omega}{\mu_0}}(1-i)^{-1} \quad (6)$$

The form on the right shows the low frequency limit, with real part $V_{phs}(\omega) = \sqrt{2\omega\rho/\mu_0}$.

Figure 1 shows $V_{phs}(\omega)$, as a function of cyclical frequency $f=\omega/2\pi$, for a case typical of the earth's sedimentary crust. It is clear that the frequency-dependence of the phase velocity may be divided into two regimes:

- a high-frequency regime, with constant phase velocity, reduced from the speed of light c by the index of refraction n (corresponding to the dominance of the second term in Eqn. (4)); this is the "displacement" regime, with geophysical application in Ground Penetrating Radar.
- a low-frequency regime, with phase velocity linear in the log-log plot, where the governing physical parameter is instead the electrical resistivity ρ **.** This is the "conduction" regime, for deep-earth investigation. Of course, low frequency is necessary for deep penetration, since the imaginary part of Eqn. (6) results in attenuation of the high frequencies.

Remarkably, at the lowest frequencies shown, the phase velocities are several km/sec, comparable to the speed of sound; the group velocities are twice as high.

At low frequency, the second term of Eqn. (4) may be neglected entirely, leaving the diffusion equation, instead of the wave equation. In this low-f limit, the EM plane wave (5) may be written as:

$$\vec{E}(\vec{x},\omega) = \vec{E}_0\,e^{i\omega(t - x/V_{phs})}\,e^{-\omega x/V_{phs}} \quad (7)$$

The EM velocity $V_{phs}(\omega)$ is highly dispersive, whereas the seismic velocity v_{phs} in Eqn. (3) is weakly dispersive. Nonetheless, the similarity of form between Equations (3)

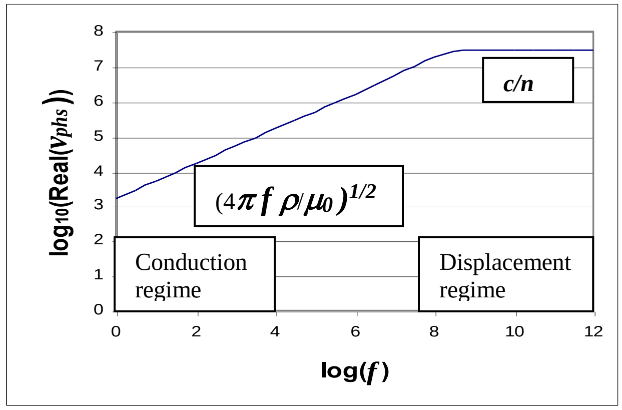

Figure 1. $V_{phs}(\omega)$, with n= 10, ρ = 1 Ohm-m.

and (7) is remarkable, revealing a deep unity between anelastic seismics and electromagnetics.

The last term in Equation (7), compared with that of Eqn. (3), lacks a factor $1/2Q_{seis}$ in the exponent; this is equivalent to defining an electromagnetic quality parameter Q_{EM} = ½. On the one hand, this is much smaller than is typical for Q_{seis}, meaning much higher attenuation. On the other hand, it is a rational fraction (intrinsic to the diffusion equation), rather than a physical property of the medium to be determined from the data, and so one may correct for it aggressively (Thomsen, et al (2009)).

This similarity between the basis functions Eqns. (3) and (7) means that *any* seismic processing algorithm which does *not* assume low dispersion and attenuation may be used to process properly acquired EM data; this is the majority of the seismic toolkit. Several illustrations of this principle are given by Neese and Thomsen (2014).

(Because of the high attenuation, EM data decreases very rapidly with source-receiver offset. This may be addressed, with trace normalization, or by other amplification techniques well known in seismics, rather than by inverting for "effective resistivity", EM-style.)

This conclusion has implications for the way controlled-source EM data is acquired. First, the receiver array must provide spatially non-aliased data, *i.e*. with smaller receiver intervals than is common today. Most importantly, in seismology we learned long ago that, since the signal from the subsurface is very weak, it should be detected *without interference* from a concurrently active source. This learning may be applied to EM data: the source should be *impulsive*-source, rather than *continuous*-source. We may call this ISEM, and interpret the acronym "CSEM" as "Continuous-Source" EM (since both are "Controlled").

DOI http://dx.doi.org/10.1190/segam2014-0798.1

The impulsive source may be realized in the field as an abrupt step (up or down) in voltage at the source antenna, followed by a long dwell time (to allow recording of the transient signal while the source is inactive) before stepping the voltage with opposite polarity at the next source-point. Since the equation is linear, one may recover an impulsive source from this step-source raw data by simple time-differentiation. If high-frequency artifacts are created by this differentiation, they may be eliminated (as un-physical) by low-pass filtering the differentiated data.

This understanding complements previous arguments regarding "time-domain CSEM", or "t-CSEM" (e.g. Strack, 1999), by offering the possibility to measure the moveout of ISEM data (Thomsen, et al, (2007, 09), Strack, et al (2008)) across the array of receivers. This moveout is a *primary observable, not available* in conventional CSEM acquisition. Since the EM phase velocity $\boldsymbol{V_{phs}}$ is comparable to the speed of sound, the "air wave" is not a problem, even if it has large amplitude. Since it is impulsively sourced, and moves out so rapidly, it is easily distinguished from the subsurface signal. Similarly, the arrival from a subsurface reservoir is also distinguished by its high velocity (see below), in addition to its amplitude.

An implication of the foregoing is that EM data may be *imaged*, seismic-style, rather than inverted mathematically. Imaging is less sensitive to variations in source strength, such as may be caused by navigational issues in EM acquisition, so it may be more robust. Since EM is intrinsically of low spatial resolution, simple imaging algorithms (*e.g.* CMP stacking) may be as effective as more elaborate wavefield algorithms (*e.g*. RTM).

The Magnetic field

Unlike with seismics, there is an associated field, the magnetic field $\vec{\boldsymbol{H}}$, which obeys an equation identical to Eqn. (4). For a single Fourier plane-wave component traveling in a macroscopically uniform isotropic medium, it is easy to show that the electric and magnetic fields travel together, and that:

- the two vector fields are always transverse to each other and to the direction of propagation (as with light in vacuum),
- the magnetic field lags the electric field by 45º (rather than by 90º, as with light in vacuum),
- the ratio of magnetic-to- electric field strengths is proportional to the local phase velocity, independent of sourcing mode or of travel distance.

When real data fail to observe these properties, it means that one or more of the assumptions is not valid, for example that more than one plane wave is contributing to the observed signal. This feature of plane waves may have some utility in exploration, for example in characterizing the nature of the arrivals at the receiver array.

EM waves at a Planar Interface

This classic problem is solved in every elementary EM textbook for special cases, such as a vacuum on one side of the interface. However, the more general problem, for arbitrary incidence on an interface with isotropic dielectrics on both sides is less common, and some disagreement exists in the literature, regarding the boundary conditions to be met. A clear discussion of the issues is given, however, in Feynman, et al (1964), where the boundary conditions are derived from the equations themselves. For non-magnetic, non-metallic (no unbound charges) materials, the boundary conditions to be met by the $\vec{\boldsymbol{E}}$ and $\vec{\boldsymbol{H}}$ fields are continuity of:

- normal components of the vector $\varepsilon\,\vec{\boldsymbol{E}}$; (8a)
- tangential components of $\vec{\boldsymbol{E}}$; (8b)
- all components of $\vec{\boldsymbol{H}}$. (8c)

Here $\varepsilon = \boldsymbol{n}^2 / \boldsymbol{c}^2 \mu_0$ is the material dielectric constant. Eqn. (8a) is different from the others because the interface presents the possibility that a surface charge could arise from a discontinuity in the field $\varepsilon\vec{\boldsymbol{E}}$. There is no similar issue for the tangential components of $\vec{\boldsymbol{E}}$. There is no similar issue for $\vec{\boldsymbol{H}}$, since there are no magnetic charges.

These equations need only be solved for incident plane waves (see Figure 2); all solutions for other contexts can be obtained by linear sums of these. Just as in the seismic case, solutions are found only if there exist both reflected (subscript 1) and transmitted (subscript 2) plane waves, with the same frequency $\boldsymbol{\omega}$, and the same horizontal component of wave vector as the incident plane wave.

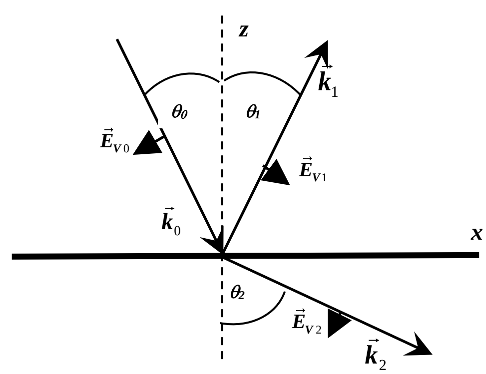

Figure 2. Geometry for reflectivity problem.

DOI http://dx.doi.org/10.1190/segam2014-0798.1

The reflected and transmitted amplitudes depend upon both the incident angle (as in seismic AVO), and the polarization of the incident wave (as in shear seismics). Since $\vec{E}_V$ (polarized in the plane, and named by analogy with the shear mode SV) is the mode excited by a conventional (inline horizontal dipole) 2D marine CSEM experiment, this case is considered here. Of course, there is no partial conversion to longitudinal polarization (as in the corresponding SV-P problem in seismics), because all EM waves are transversely polarized.

Consider Figure 2, with an ***xy*** interface, and with incidence in the ***xz*** plane. The ***x***-component of the wave vectors are:

$$\boldsymbol{k}_x = \boldsymbol{k}_0 \sin\theta_0 = \boldsymbol{k}_1 \sin\theta_1 = \boldsymbol{k}_0 \sin\theta_2 ; \tag{9}$$

this is just Snell's law, as in seismics, except that the wave vectors are complex. The real parts are:

$$\frac{\omega}{\boldsymbol{V}_{phs0}} \sin\theta_0 = \frac{\omega}{\boldsymbol{V}_{phs0}} \sin\theta_1 = \frac{\omega}{\boldsymbol{V}_{phs2}} \sin\theta_2 \tag{10}$$

so that the incidence and reflected angles are equal (as in seismics), and the transmitted angle is given by:

$$\sin\theta_2 = \left(\boldsymbol{V}_{phs2} / \boldsymbol{V}_{phs0}\right) \sin\theta_0 = \sqrt{\rho_2 / \rho_0}\, \sin\theta_0 \tag{11}$$

with a critical angle (beyond which the transmission angle is complex) given by:

$$\sin\theta_{0Crit} = \left(\boldsymbol{V}_{phs0} / \boldsymbol{V}_{phs2}\right) = \sqrt{\rho_0 / \rho_2} \tag{12}$$

In the marine EM exploration context, there are three interfaces of primary interest:

- the water → air interface, with critical angle:

$$\sin\theta_{0Crit} = \sqrt{\rho_0 / \rho_2} \approx \sqrt{.33/\infty} \approx 0 \tag{13a}$$

- the water → mud interface, with critical angle:

$$\sin\theta_{0Crit} \approx \sqrt{.33/1} \Rightarrow \theta_{0Crit} \approx 35^o \tag{13b}$$

- the overburden → reservoir interface, with critical angle:

$$\sin\theta_{0Crit} \approx \sqrt{1/100} \Rightarrow \theta_{0Crit} \approx 6^o \tag{13c}$$

Eqn. (13a) means that energy upcoming from a deep-towed source refracts horizontally (the "air wave"), even for tiny angles of incidence. Eqn. (13c) means that energy down-going to a reservoir layer refracts horizontally, even for quite small angles of incidence.

Similarly, k_z in each medium is given by

$$\boldsymbol{k}_{zi} = \boldsymbol{k}_i \cos\theta_i = \left(\omega / \boldsymbol{V}_{phsi}\right) \sqrt{1 - \sin^2\theta_i} \tag{14}$$

From Feynman, et al (1964), the amplitudes are:

$$\frac{\boldsymbol{E}_{V1}}{\boldsymbol{E}_{V0}} = \frac{\boldsymbol{V}_{phs0}^2 \boldsymbol{k}_{z0} - \boldsymbol{V}_{phs2}^2 \boldsymbol{k}_{z2}}{\boldsymbol{V}_{phs0}^2 \boldsymbol{k}_{z0} + \boldsymbol{V}_{phs2}^2 \boldsymbol{k}_{z2}} \text{ …reflected wave} \tag{15a}$$

$$\frac{\boldsymbol{E}_{V2}}{\boldsymbol{E}_{V0}} = \frac{2\boldsymbol{V}_{phs0} \boldsymbol{V}_{phs2} \boldsymbol{k}_{z0}}{\boldsymbol{V}_{phs0}^2 \boldsymbol{k}_{z0} + \boldsymbol{V}_{phs2}^2 \boldsymbol{k}_{z2}} \text{ …refracted wave} \tag{15b}$$

Eqns. (15) are complex, but independent of frequency (in this half-space problem). At low frequencies, from (14, 15):

$$\frac{\boldsymbol{E}_{V1}}{\boldsymbol{E}_{V0}} = \frac{\sqrt{\rho_0/\rho_2}\cos\theta_0 - \cos\theta_2}{\sqrt{\rho_0/\rho_2}\cos\theta_0 + \cos\theta_2} \text{ … reflected wave} \tag{16a}$$

$$\frac{\boldsymbol{E}_{V2}}{\boldsymbol{E}_{V0}} = \frac{2\cos\theta_0}{\sqrt{\rho_0/\rho_2}\cos\theta_0 + \cos\theta_2} \text{ …refracted wave} \tag{16b}$$

Comparing the refracted and reflected modes:

$$\frac{\boldsymbol{E}_{V2}}{\boldsymbol{E}_{V1}} = \frac{2\cos\theta_0}{\sqrt{\rho_0/\rho_2}\cos\theta_0 - \cos\theta_2} >> 1 \text{ if } \boldsymbol{\rho_2} >> \boldsymbol{\rho_1} \tag{16c}$$

Hence, at the top of the reservoir, *most* of the incident energy is refracted; if it is recorded at the surface, it has the apparent velocity of the *reservoir*. Since travel within a reservoir is always faster; a reservoir may potentially be detected by fast *moveout* across the receiver array (*e.g.* Thomsen, et al (2009), Neese and Thomsen (2014)). Further, since the velocity is faster, the wavelength of each frequency is longer, and the wave executes fewer cycles per km, so the attenuation is less, and the amplitude is higher.

By contrast, in reflection seismics the moveout velocity is controlled by the overburden, and the properties of the thin reservoir are encoded only in the reflection amplitude.

Conclusions

Seismic and EM data have deep mathematical connections, which lead to strong implications for EM data acquisition, processing, and interpretation:

- Both may be described as a superposition of dispersive, attenuative plane waves.
- Hence any seismic data-processing algorithm which does not assume weak dispersion or attenuation may be applied to EM data, properly acquired.
- Like seismic data, EM data are best acquired with an impulsive source ("ISEM") (so that the weak signal from the subsurface is not dominated by a source continuously broadcasting ("CSEM")), and recorded without spatial aliasing.
- With ISEM, EM moveout (several km/s) is a primary observable, and may be used to detect and to characterize hydrocarbon reservoirs.

All of these theoretical conclusions are confirmed (Neese and Thomsen, 2014), by forward modelling of a simple 1D case (using EM methods), followed by data-processing (using seismic methods) as suggested here.

Acknowledgement
The author thanks Kurt Strack for bringing these issues to his attention, and for many useful conversations.

http://dx.doi.org/10.1190/segam2014-0798.1

EDITED REFERENCES

Note: This reference list is a copy-edited version of the reference list submitted by the author. Reference lists for the 2014 SEG Technical Program Expanded Abstracts have been copy edited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

REFERENCES

Feynman, R. P., R. B. Leighton, and M. Sands, 1964, The Feynman lectures on physics II: Addison Wesley Press.

Futterman, W. I., 1962, Dispersive body waves: Journal of Geophysical Research, **67**, no. 13, 5279–5291, http://dx.doi.org/10.1029/JZ067i013p05279.

Jackson, J. D., 1962, Classical electromagnetics: Wiley Press.

Neese, J. W., and L. Thomsen, 2014, Seismic processing of numerical EM data: Presented at the 84th Annual International Meeting, SEG.

O'Connell, R. J., and B. Budianski, 1978, Measures of dissipation in viscoelastic media : Geophysical Research Letters, **5**, no. 1, 5–8, http://dx.doi.org/10.1029/GL005i001p00005.

Strack, K. M., Exploration with deep transient electromagnetics: Elsevier Press.

Strack, K. M., N. Allegar, and S. Ellingsrud, 2008, Marine time domain CSEM: An emerging technology: 78th Annual International Meeting, SEG, Expanded Abstracts, 653–656.

Thomsen, L. A., N. C. Allegar, J. A. Dellinger, P. Jilek, D. T. Johnson, and G. Xia, 2009, System and method for using time-distance characteristics in acqusition, processing, and imaging of t-CSEM Data: U. S. Patents 7,502,690 and 7941273.

Thomsen, L. A., D. Meaux, S. Li, C. Weiss, A. Sharma, N. Allegar, and K. Strack, 2007, Novel marine electromagnetics: From deep into shallow water: Recent advances and the road ahead: Presented at the 77th Annual International Meeting, SEG.

Ursin, B., 1983, Review of elastic and electromagnetic wave propagation in horizontally layered media : Geophysics, **48**, 1063–1081, http://dx.doi.org/10.1190/1.1441529.