On the fluid dependence of the parameters of anisotropy

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Summary

The theory for the fluid dependence of the elastic compliance of general anisotropic formations was given by Brown and Korringa (1975), but in terms that are difficult to understand intuitively. Those equations are recast here, leading to new expressions (Eqns. (11, 12) below) for the fluid dependence of the standard parameters (for the case of weak polar anisotropy) $\delta, \varepsilon, \gamma$. They have a broad similarity to the well-known isotropic expression for the fluid dependence of incompressibility, and assert that both γ and $(\varepsilon-\gamma)$ are independent of fluid content.

Introduction

The generally accepted theory governing the effect of fluid content on seismic velocities is due to Biot (1941) and Gassmann (1951), with further refinement by Brown and Korringa (1975) ("BK75"), and most recently by Thomsen (2010b). Most applications of the theory use its isotropic special case, and without the refinements by BK75 and Thomsen (2010b). The present work concerns the extension of this theory to the case of anisotropy.

In fact, the case of polar anisotropy (previously named Transverse Isotropy [*sic!*]) was given by Gassmann (1951). BK75 treated the general (homogeneous) anisotropy, but in notation that makes it difficult to answer the question: what is the prediction of the theory for the fluid dependence of the nondimensional anisotropic parameters, e.g. $\delta, \varepsilon, \gamma$ (Thomsen, 1986). The present work presents explicit expressions for these dependencies, for the case of weak polar anisotropy.

Theoretical Background

It is well-known that the isotropic form of Biot-Gassmann theory predicts *neither* the dry elastic moduli of a rock, nor its saturated moduli, but only the *difference* in these two. The theory concludes that for the shear modulus, this difference is zero, and for the bulk modulus is given by the explicit formula:

$$K_{sat}\left(K_{fld}\right) - K_{dry} = \frac{\alpha^2}{\left[\phi\left(\frac{1}{K_{fld}} - \frac{1}{K_{sol}}\right) + \frac{\alpha}{K_{sol}}\right]} \quad (1)$$

where K_{sat}, K_{dry}, K_{fld}, K_{sol} are the <u>incompressibilities</u> of (respectively): the fluid-saturated rock, the rock with gas-filled pore space, the pore-filling fluid, and the solid grains of the rock. ϕ is porosity, and $\alpha = 1 - K_{dry}/K_{sol}$. This is exactly equivalent to the following expression in terms of compressibilities $\kappa = 1/K$:

$$\kappa_{sat}\left(\kappa_{fld}\right) - \kappa_{dry} = -\frac{\alpha^2 \kappa_{dry}^2}{\left[\phi\left(\kappa_{fld} - \kappa_{sol}\right) + \alpha\kappa_{dry}\right]} \quad (2)$$

Equation (2) is derived by BK75, who also conclude that both (1) and (2) are strictly valid only for a homogeneous solid. (For a heterogeneous solid, they derive a slightly different formula. Thomsen (2010) argued that this variant is required even in the case of a homogeneous solid, but for simplicity, we ignore this complication here.)

BK75 also derive the anisotropic generalization of (2):

$$S_{\alpha\beta}^{sat} - S_{\alpha\beta}^{dry} = -\frac{\left(S_{\alpha}^{dry} - S_{\alpha}^{sol}\right)\left(S_{\beta}^{dry} - S_{\beta}^{sol}\right)}{D} \quad (3a)$$

where $S_{\alpha\beta}$ is an element of the 4th –rank elastic compliance tensor, expressed in Voigt notation,

$$S_{\alpha} \equiv \sum_{\beta=1}^{3} S_{\alpha\beta} \tag{3b}$$

and

$$D \equiv \varphi(\kappa_{fld} - \kappa_{sol}) + \alpha\kappa_{dry} \tag{3c}$$

While equations (3) provide a *formal* solution to the present problem, they do not provide an *intuitive* answer to the question of the fluid dependence of the parameters which control the anisotropic variation of seismic velocities. For that, further development is required; the following is restricted to the case of polar anisotropy, but can be extended in straightforward fashion to lower symmetries, e.g. to orthorhombic or monoclinic fracture systems.

Shale

It is necessary to recognize explicitly that shales differ geophysically from other lithologies, not only in their greater seismic anisotropy, but also in their lower hydraulic permeability. This means that the assumption (common to this work, Biot (1941), Gassman (1951), BK75, Thomsen (2012b) and most other low-frequency studies of fluid dependence), that (locally, on the pore scale) the fluid pressure is *uniform*, may not be valid. This petrophysical issue is outside the scope of the present seismic discussion; we start with Eqns. (3), which incorporate this common assumption.

Layered anisotropy

Equations (1-3) above concern homogeneous rock masses. It is well-known that thin-layer sequences (whether composed of isotropic or anisotropic layers) result in long-wavelength seismic anisotropy (Backus, 1962). That is, if the statistics of the layering are stationary, the sequence is effectively (at long wavelength) homogeneous and anisotropic. In order to theoretically compute the fluid dependences of the anisotropic parameters of such a formation, it is necessary to compute *separately* the fluid dependences for each individual layer (assumed homogeneous), then combine them according to the expressions given by Backus, 1962. Then one computes the consequent anisotropy parameters (Thomsen, 1986), and finally compares with other fluid conditions. The following discussion refers to the intrinsically anisotropic layers within the sequence, or to thick anisotropic formations, *e.g*. to massive shales.

Shear anisotropy

As noted by BK75, both polar anisotropic shear compliances (S_{44} and S_{66}) are easily understood, since (cf. Eqn. (3b)) $S_4 = S_6 = 0$. From Eqn. (3a), it follows that both S_{44} and S_{66} are independent of fluid content. Since the shear stiffnesses C_{44} and C_{66} are simply the inverses of the corresponding compliances, it follows that both C_{44} and C_{66} are independent of fluid content. Hence, the shear anisotropy parameter γ is likewise independent of fluid content. The rest of this work concerns the P/S parameters δ and ε.

Weakly anisotropic elastic compliance

Formally, the elastic compliance tensor $\tilde{S}$ is the inverse of the elastic stiffness tensor $\tilde{C}$:

$$C_{ijkl} S_{klmn} = I_{ijmn} \equiv \frac{1}{2}\left(\delta_{im}\delta_{jn} + \delta_{in}\delta_{jm}\right) \tag{4}$$

If we consider perturbations $\Delta\tilde{C}$ and $\Delta\tilde{S}$ from any assumed base case, then it is easy to show (*c.f.* Sayers, 2009) that, to first order in the perturbations,

$$\Delta S_{pqmn} = -S^0_{pqij} \Delta C_{ijkl} S^0_{klmn} \tag{5}$$

Consider that the base case is an isotropic body with elastic stiffnesses and compliances given by (in Voigt notation, with only the upper triangle shown, and the zeroes suppressed):

$$\tilde{C}^0 = \begin{vmatrix} M_3 & \lambda_3 & \lambda_3 & & & \\ & M_3 & \lambda_3 & & & \\ & & M_3 & & & \\ & & & \mu_3 & & \\ & & & & \mu_3 & \\ & & & & & \mu_3 \end{vmatrix} \tag{6a}$$

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$$\tilde{S}^0 = \begin{vmatrix} \frac{1}{E_3} & \frac{-\nu_3}{E_3} & \frac{-\nu_3}{E_3} & & & \\ & \frac{1}{E_3} & \frac{-\nu_3}{E_3} & & & \\ & & \frac{1}{E_3} & & & \\ & & & \frac{1}{\mu_3} & & \\ & & & & \frac{1}{\mu_3} & \\ & & & & & \frac{1}{\mu_3} \end{vmatrix} \qquad (6b)$$

where $M_3 = K_3 + 4\mu_3/3$, μ_3, λ_3, E_3 are its longitudinal, shear, Lame, and Young's moduli respectively, and ν_3 is its Poisson's ratio, all connected via the standard isotropic expressions. The subscript 3 is used to indicate that these are chosen to match *exactly* the anisotropic parameters in the 3-direction, i.e. $M_3 = C_{33}$; $\mu_3 = C_{44}$; $1/E_3 = S_{33}$ etc., so that $\Delta C_{3333} = \Delta C_{3131} = \Delta C_{3232} = 0$, etc.

Then Eqn. (5) leads to expressions for the anisotropic variation of compliances in terms of $\delta, \varepsilon, \gamma$; these are the same as those given by Thomsen (2010a), using a less-elegant algebraic approach:

$$\Delta S_{11} = -\left(\frac{M_3(1-\nu_3)}{E_3^2}\right)\left[\left(1-\nu_3\right)2\varepsilon - 2\nu_3\delta + \frac{8\nu_3\mu_3}{M_3}\gamma\right] \qquad (7a)$$

$$\Delta S_{33} = -\left(\frac{M_3}{E_3^2}\right)\left[8\nu_3^2\varepsilon - 4\nu_3\delta - \frac{8\nu_3^2\mu_3}{M_3}\gamma\right] \qquad (7b)$$

$$\Delta S_{13} = -\left(\frac{M_3\nu_3\left(1-\nu_3\right)}{E_3^2}\right)\left[-4\varepsilon + \frac{(1-\nu_3+2\nu_3^2)}{\nu_3\left(1-\nu_3\right)}\delta + 4\gamma\frac{\mu_3}{M_3}\right] \qquad (7c)$$

$$\Delta S_{12} = \Delta S_{11} - \frac{1}{2}\Delta S_{66} \qquad (7d)$$

Equations (7) apply to any saturation state, or to the solid grains (with appropriate superscripts). With the assumption of weak polar anisotropy, Eqn. (3a) becomes:

$$-D_0\left(\Delta S_{\alpha\beta}^{sat} - \Delta S_{\alpha\beta}^{dry}\right) = \left(D - D_0\right)\left(S_{\alpha\beta}^{0sat} - S_{\alpha\beta}^{0dry}\right) + \left(S_{\alpha}^{0dry} - S_{\alpha}^{0sol}\right)\left(\Delta S_{\beta}^{dry} - \Delta S_{\beta}^{sol}\right) + \left(S_{\beta}^{0dry} - S_{\beta}^{0sol}\right)\left(\Delta S_{\alpha}^{dry} - \Delta S_{\alpha}^{sol}\right) \qquad (8a)$$

where

$$S_{\alpha}^{0} = \frac{1}{3K_3} \qquad (8b)$$

for all α, and

$$D_0 \equiv \left(\frac{1}{K^{fld}} - \frac{1}{K_3^{sol}}\right)\phi + \left(\frac{1}{K_3^{dry}} - \frac{1}{K_3^{sol}}\right) \qquad (8c)$$

Implementing Eqns. (8) separately for matrix elements 11, 33, 13, and 12, and combining these yields:

$$(\Delta S_{33}^{sat} - \Delta S_{11}^{sat}) - (\Delta S_{33}^{dry} - \Delta S_{11}^{dry}) = \frac{-2}{3D_0}\left(\frac{1}{K_3^{dry}} - \frac{1}{K_3^{sol}}\right)\left((\Delta S_3^{dry} - \Delta S_1^{dry}) - (\Delta S_3^{sol} - \Delta S_1^{sol})\right) \qquad (9a)$$

$$(\Delta S_{13}^{sat} - \Delta S_{12}^{sat}) - (\Delta S_{13}^{dry} - \Delta S_{12}^{dry}) = \frac{-1}{3D_0}\left(\frac{1}{K_3^{dry}} - \frac{1}{K_3^{sol}}\right)\left((\Delta S_3^{dry} - \Delta S_3^{sol}) - (\Delta S_2^{sol} - \Delta S_2^{sol})\right) \qquad (9b)$$

Observe that forming the anisotropic combinations (ΔS_{33} - ΔS_{11}) and (ΔS_{13} - ΔS_{12}) simplifies these expressions by eliminating the term in (D-D_0) from Eqn. (8a). Note that the right side of Eqn. (9b) is exactly ½ that of Eqn. (9a).

Substituting Eqns. (7) into equations (9) gives two equations in two unknowns, $(\varepsilon^{sat} - \varepsilon^{dry})$ and $(\delta^{sat} - \delta^{dry})$, which may be re-arranged to give:

$$\left[M_3^{sat}\varepsilon^{sat} - M_3^{dry}\varepsilon^{dry}\right] = \left[M_3^{sat}\delta^{sat} - M_3^{dry}\delta^{dry}\right] \qquad (10)$$

and

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$$\left(\frac{M_3^{sat}}{3K_3^{sat}\mu_3}\right)\varepsilon^{sat} - \left(\frac{M_3^{dry}}{3K_3^{dry}\mu_3}\right)\varepsilon^{dry} = \frac{-2\alpha}{3D_0 K_3^{dry}}\left[(\Delta S_3^{dry} - \Delta S_1^{dry}) - (\Delta S_3^{sol} - \Delta S_1^{sol})\right] + \left(\frac{\nu_3^{sat}}{E_3^{sat}} - \frac{\nu_3^{dry}}{E_3^{dry}}\right)\left(\frac{M_3^{dry}}{\mu_3}(\varepsilon^{dry} - \delta^{dry}) - 4\gamma\right) \tag{11a}$$

where

$$\left[(\Delta S_3^{dry} - \Delta S_1^{dry}) - (\Delta S_3^{sol} - \Delta S_1^{sol})\right] = \left[\frac{M_3^{dry}}{6K_3^{dry}\mu_3^{dry}}\left(4\varepsilon^{dry} - \delta^{dry}\right) - \frac{M_3^{sol}}{6K_3^{sol}\mu_3^{sol}}\left(4\varepsilon^{sol} - \delta^{sol}\right)\right] - \frac{2}{3}\left[\frac{\gamma^{dry}}{K_3^{dry}} - \frac{\gamma^{sol}}{K_3^{sol}}\right] \tag{11b}$$

If the variation due to fluid substitution is small, these simplify considerably:

$$(\varepsilon^{sat} - \varepsilon^{dry}) \approx (\delta^{sat} - \delta^{dry}) \tag{12a}$$

$$(\varepsilon^{sat} - \varepsilon^{dry}) \approx \frac{-2\alpha}{D_0}\left(\frac{M_3^{dry}}{\mu_3}\right)^{-1}\left[(\Delta S_3^{dry} - \Delta S_1^{dry}) - (\Delta S_3^{sol} - \Delta S_1^{sol})\right] \tag{12b}$$

If the solid grains are isotropic, then Eqn. (11b) simplifies further to:

$$\left[(\Delta S_3^{dry} - \Delta S_1^{dry}) - (\Delta S_3^{sol} - \Delta S_1^{sol})\right] = (\Delta S_3^{dry} - \Delta S_1^{dry}) = \frac{M_3^{dry}}{6K_3^{dry}\mu_3^{dry}}\left(4\varepsilon^{dry} - \delta^{dry}\right) - \frac{2\gamma^{dry}}{3K_3^{dry}} \tag{13}$$

and this simpler expression also simplifies Eqn. (12).

Discussion and conclusions

Recall the (seismically determinable) abnormal moveout parameter introduced by Alkhalifah and Tsvankin (1995):

$$\eta \equiv \frac{\varepsilon - \delta}{1 + 2\delta} \approx \varepsilon - \delta \tag{14}$$

It is remarkable that, in this approximation, from Eqn. (12a), we find that

$$\eta^{sat} \approx \eta^{dry} \tag{15}$$

Hence, to this approximation, η (like γ) is invariant to fluid substitution. Intuitively, one can say that both ε and δ depend upon the compressibility of the pore fluid, in an identical way, so that their *difference* is invariant with respect to fluid compressibility.

That dependence is given by Eqn. (12b), which is an explicit expression for the dependence of both parameters ε and δ on saturation state, in terms of an expression, on the right, which has a structure similar to that of Eqn. (2), depending only on the parameters of the dry state, and of the solid, and of the fluid and porosity (inside D_0).

The more exact equations (11) differ from these by amounts which depend upon the additional terms evident there. Since E_3^{sat} appears on the right side, an iterative solution is recommended.

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