On the Fluid Dependence of Rock Compressibility: Biot-Gassmann Refined

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Introduction

An important issue in using seismic data to physically characterize crustal rocks is the dependence of seismic rock velocity upon the compressibility of the fluid in the pore space. At seismic frequencies, the standard theory for understanding this fluid dependence is that of Biot (1941) and Gassmann (1951). It is well-known that this theory predicts that the rigidity should be independent of fluid content, and that the fluid dependence of the incompressibility is given by an explicit formula (Eqn. (1), below). The purpose of this note is to point out an approximation in the theory that has not been fully appreciated. Below it is shown that avoiding this approximation leads to a refinement of the standard Biot-Gassmann ("B-G") formula for fluid substitution in rock incompressibility, with new requirements (compressibility data) for implementation. This revision has been reported previously (Brown and Korringa, 1975) in a limited context (non-uniform mineralogy); here it is shown to be much more general. The refined formula fits experimental data better than does the classic formula.

Experimental considerations

It is sobering to reach the conclusion that a theory that has been well-accepted for over half a century is in fact *not* correct. In such a case, we should immediately re-examine the experimental verification of the theoretical predictions. It turns out that the theory has *not*, in fact, had strong experimental verification, despite its wide acceptance. The ambiguous state of the experimental test of the B-G conclusions was discussed *e.g.* by Thomsen (1986), Winkler (1986), and Z. Wang (2000); this work is not reviewed here.

A principal difficulty is that most of the previous experimental approaches have measured elastic velocities (P and S) at ultrasonic frequencies (in gas-saturated and liquid-saturated rock), whereas the theory is explicitly limited to quasi-static (or low-frequency) measurements, with the assumption that it applies to elastic velocities at seismic frequencies. Of course, the reason for this indirect experimental strategy is that direct measurements of compressibility under quasi-static conditions are difficult at the low strains ($<10^{-6}$) beyond which non-linear effects become important. The following analysis shows that in fact static compressibility tests are *required* in order to fully specify the parameters in the (refined) theory; recent work by Hart and Wang (2010) enables a fresh assessment.

Theoretical considerations

The proof of the Biot-Gassmann formula is here recapitulated, following the compact and clear argument of Brown and Korringa (1975) ("B&K"), but with an important change. The argument is conducted in terms of compressibility rather than of incompressibility, but leads eventually to the classic low-frequency B-G formula:

$$K_{sat}\left(K_{fld}\right) = K_{dry} + \frac{K_{fld}\,\alpha^2}{\left[\phi + \dfrac{K_{fld}}{K_{sol}}(\alpha - \phi)\right]} \qquad (1)$$

where K_{sat} , K_{dry} , K_{fld} , K_{sol} are the <u>incompressibilities</u> of (respectively): the fluid-saturated rock, the rock with completely empty pore space, the pore-filling fluid, and the solid grains of the rock. ϕ is porosity, and $\alpha = 1 - K_{dry}/K_{sol}$.

Using mainly just calculus, B&K derive the difference between the dry compressibility κ_A and the saturated compressibility κ* (their Eqn. (13)):

$$\kappa_A - \kappa^* = \frac{(\kappa_A - \kappa_M)\kappa'}{[(\kappa_F - \kappa_\phi)\phi + \kappa']} \tag{2}$$

in terms of various measures of compressibility of a mass element ("undrained") κ_M, κ', κ_F, κ_ϕ defined by them. In particular, the "unjacketed compressibility" κ_M and the "pore compressibility" κ_ϕ are:

$$\kappa_M \equiv \frac{-1}{V}\left(\frac{\partial V}{\partial p_F}\right)_{p_d} \tag{3a}$$

$$\kappa_\phi \equiv \frac{-1}{V_\phi}\left(\frac{\partial V_\phi}{\partial p_F}\right)_{p_d} \tag{3b}$$

where V is the volume of the mass element, $V_\phi=\phi V$ is its pore volume, p_F is the fluid pressure in the pore space, p is the total ("confining") pressure, and $p_d=p-p_F$ is the differential pressure. Using the Reciprocity Theorem (Love, 1944), B&K obtained $\kappa' = \kappa_A - \kappa_M = \kappa_A \alpha_M$, leading to their primary scalar result:

$$\kappa_A - \kappa^* = \frac{(\kappa_A - \kappa_M)^2}{[(\kappa_F - \kappa_\phi)\phi + (\kappa_A - \kappa_M)]} = \frac{\alpha_M^2 \kappa_A^2}{[(\kappa_F - \kappa_\phi)\phi + \alpha_M \kappa_A]} \tag{4}$$

They remark upon their success in generalizing the B-G result at the cost of only one additional parameter (κ_ϕ), but do not emphasize that the appearance in (4) of κ_M (instead of the solid compressibility κ'') complicates the implementation substantially. However, for a rock with homogeneous solid, B&K concluded (following arguments discussed below) that, $\kappa_M = \kappa_\phi = \kappa''$ (the compressibility of the solid grains), so that Eqn. (4) becomes

$$\kappa_A - \kappa^* = \frac{(\kappa_A - \kappa'')^2}{[(\kappa_F - \kappa'')\phi + (\kappa_A - \kappa'')]} = \frac{\alpha^2 \kappa_A^2}{[(\kappa_F - \kappa'')\phi + \alpha \kappa_A]} \tag{5}$$

It is easy to show that Eqn. (5) is exactly the same as the classical B-G result (1), written in this notation.

Here, we concentrate on B&K's conclusion that $\kappa_M = \kappa_\phi = \kappa''$ for monomineralic rocks. This conclusion follows from their assertion that equality of stress across all the solid-fluid surfaces Σ_ϕ of the rock results in a self-similar volume change, with no change in porosity (pore volume/total volume). In fact, it is easy to show that the change in porosity is given by

$$\left(\frac{\partial \phi}{\partial p_F}\right)_{p_d} = \phi(1-\phi)(\kappa_M - \kappa_\phi) \tag{6}$$

and that $\kappa_M = \frac{-1}{V}\left(\frac{\partial (V_M + V_\phi)}{\partial p_F}\right)_{p_d} = (1-\phi)\kappa'' + \phi\kappa_\phi = \kappa'' + \phi(\kappa_\phi - \kappa'')$ (7)

from which it is easy to see that invariance of ϕ leads to the conclusion that $\kappa_M = \kappa_\phi = \kappa''$.

B&K simply assert that ϕ is invariant, with respect to p_F . However that conclusion is clearly *proven* by Gurevich (2004); the proof allows examination of its assumptions. In the proof, p_F is regarded as an independent variable (to be adjusted by an experimenter), whereas in the present context, it is a *dependent* variable, determined by the external pressure, by the rock properties, and by the assumption of no fluid flow into or out of the mass element. In this undrained context, p_F is linearly related to the volume V_ϕ (through the fluid compressibility κ_F); this connection means that the deformation *cannot* be self-similar.

In the general case, the distortion of the solid by the application of uniform p_F throughout the rock is a complicated function of solid incompressibility, rigidity, and micro-geometry. But, it is easy to imagine an artificial case where none of this matters. Imagine the case where the mass element consists of a cube of homogeneous isotropic solid material with volume V_M, surrounded by a cubic shell of fluid with volume V_ϕ. (If it is desired that the solid should be simply connected, imagine very thin spacers, made of the same solid material, which connect the solid cube to other similar cubes in nearby mass elements, but which do not materially distort the stresses.) Application of uniform p_F (at constant p_d) throughout this peculiar mass element results in no shear stress at all, only a uniform compression of both solid and fluid, each according to its own compressibility (κ'' or κ_F , respectively). This changes the *relative* volume fraction occupied by solid and by fluid, hence it changes the fluid-filled porosity, $\phi = V_\phi / V$. Hence, from Eqns. (6, 7) it follows that κ_ϕ , κ_M , and κ'' are all different. Of course this artificial case has no significance in itself, but only illustrates, by example, the *principle* that in the general case, κ_ϕ , κ_M , and κ'' are all different.

This argument confirms B&K's conclusion that κ_ϕ differs from κ_M and κ'' , and generalizes it to include even the special case of homogenous solid. Hence, it is certainly necessary to consider the complications posed by the appearance in Eqn. (4) of κ_ϕ , and κ_M in even the simplest cases. In terms of incompressibilities, Eqn. (4) becomes (compare with B&K (2)):

$$K_{sat}\left(K_{fld}\right) = K_{dry} + \frac{K_{fld}\alpha_M^2}{\left[\left(1 - K_{fld}\kappa_\phi\right)\phi + \alpha_M K_{fld}\kappa_M\right]} \tag{8}$$

The difficulty in using this expression for seismic fluid substitution is that (unlike the solid compressibility κ'') both κ_ϕ and κ_M are parameters of the *rock*, and depend upon composition, rigidity, and micro-geometry in a complicated way, not susceptible to theoretical resolution.

In recent compressibility experiments, Hart and Wang (2010) independently measured all the parameters of Eqn. (4) except for κ_ϕ, in Berea sandstone and Indiana limestone, and found that (especially in the sandstone) that κ_ϕ differed from κ_M, in a stress-dependent way. (Since this approach assumes the validity of Eqn. (4), it cannot be used as a check of the same equation.) They also measured Skempton's B-coefficient, the ratio of fluid pressure to confining pressure in an undrained experiment, given in terms of compressibilities by (H. F. Wang, 2000):

$$B \equiv \frac{\delta p_F}{\delta p} = \frac{\kappa_A - \kappa^*}{\kappa_A - \kappa_M} \tag{9}$$

and used this overdetermined dataset to prove their point, conservatively. However, their data can also serve as a validity check of Eqns. (4, 8). In the course of deriving Eqn. (2), B&K analyze the variation of the pore volume with applied pressure in an undrained experiment, finding that (their Eqn. (12)):

$$-\phi\kappa_F \delta p_F = -\kappa'(\delta p - \delta p_F) - \phi\kappa_\phi \delta p_F \tag{10}$$

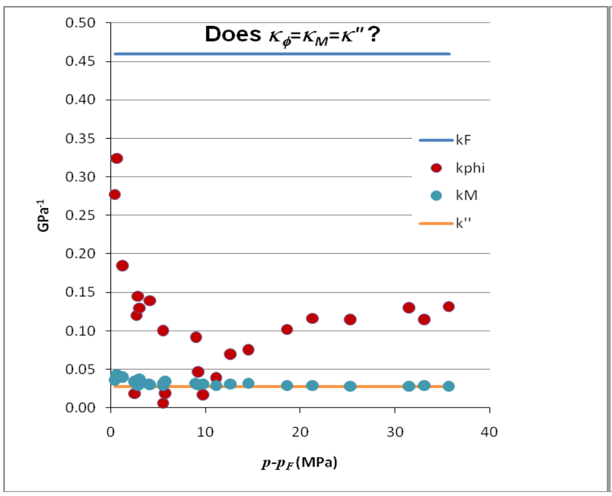
Using their subsequent result that $\kappa' = \kappa_A - \kappa_M$, this is

$$\frac{\delta p}{\delta p_F} = \frac{1}{B} = 1 + \frac{\phi(\kappa_F - \kappa_\phi)}{(\kappa_A - \kappa_M)} \tag{11}$$

so that the pore compressibility is given as

$$\kappa_\phi = \kappa_F - (\frac{1-B}{\phi B})(\kappa_A - \kappa_M) \tag{12}$$

Using the sandstone data from Hart and Wang (2010, Table 1), Fig. 1 confirms their conclusion that $\kappa_\phi \neq \kappa_M \neq \kappa''$. This conclusion seems to be firm, even with this less conservative approach (despite the evidence of scatter in the data, especially at differential pressures greater than 15 MPa). Because of the pressure-dependence of the differences, they cannot be due to the fact that the sandstone has about 20% of non-quartz minerals, but must be due to the microgeometry of the rock, in particular to the closing of microcracks with increasing pressure.



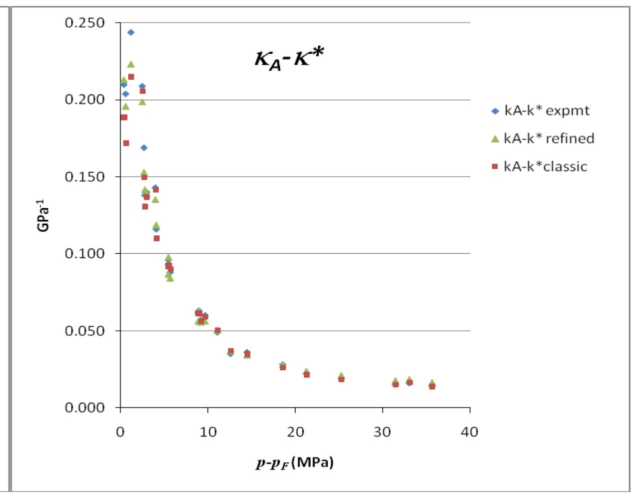


Figure 1. Demonstrating that $\kappa_\phi \neq \kappa_M \neq \kappa''$

Fig. 2. Neglect of this complexity causes systematic error.

Further, when these measured values are used to calculate the difference $\kappa_A - \kappa^*$, using the refined B-G Eqn. (4), the agreement with experiment is fairly good (Fig. 2). When the classic B-G formula (5) is used to calculate this difference, the errors are somewhat larger, and biased low, particularly at low differential pressure.

Conclusions

In the classic expression (1 or 5) of Biot-Gassmann for fluid substitution in the rock compressibility, $\kappa_A - \kappa*$ contains an approximation previously identified (by B&K, in a limited context), whose generality has not been fully appreciated. Correction of this approximation (using Eqn. (4) replacing Eqn. (5) requires data on rock compressibility. With the correction, agreement of theory and the experimental data of Hart and Wang (2010) is fairly good. By contrast, the classic equation gives predictions which are systematically lower. The significance of the correction depends upon the particular circumstances, and is most important where the rock contains cracks or fractures.

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EDITED REFERENCES

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