On the use of isotropic parameters λ, E, ν to understand anisotropic shale behaviour

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Summary

Most modern seismic data analysis of shale resource prospects is done using anisotropic methods, but much of the subsequent subsurface characterization is done using isotropic concepts, for example using the isotropic elastic parameters λ, E, and ν. This inconsistency introduces errors because of the neglected anisotropy of the shale. This work gives explicit expressions for these errors, in terms of the anisotropy parameters δ, ε, and γ, which are familiar from seismic analysis.

Introduction

Most modern *seismic* data acquisition, processing, imaging, and interpretation, which are aimed at understanding shale resource prospects, is done using anisotropic concepts and algorithms. Of course, the reason for this is that the reservoir rocks (and indeed most of the overburden rocks) are in fact anisotropic, and it is now realized that neglecting this fact can lead to poor results. The anisotropy can be simple (*i.e.* polar anisotropy for unfractured shales) or complex (*i.e.* azimuthal anisotropy (orthorhombic or monoclinic; never "HTI" or "VTI + HTI" [*sic!*]) for fractured shales.

However, for *mechanical* analysis, *e.g.* (ductility, brittle-ness, "frackability", etc. it is common to use empirical relations which rely upon *isotropic* concepts (applied to these very same anisotropic rocks!) such as the Lame parameter λ, the Young's modulus E, and/or the Poisson's ratio ν. The present work aims to understand the consequences of this inconsistent practice.

In the following, we will refer to these "isotropic parameters" of anisotropic rocks as "apparent isotropic parameters". A proper anisotropic interpretation of what they might mean depends upon 1) how they are determined, and 2) the symmetry of the anisotropic rocks. Here we will consider several classes of data which can lead to the determination of these apparent isotropic parameters; in all cases the data comes from wave propagation, and the apparent isotropic parameters are calculated from this data, using isotropic formulae. For simplicity, we will assume here that the rocks are polar anisotropic (*i.e.* that all horizontal directions are equivalent, and different from the vertical direction; this used to be called "VTI" [*sic*!], and is a plausible idealization of unfractured shales (as in Figure 1). However, these methods can be extended to lower symmetries in straight-forward fashion; *e.g.* to orthorhombic or monoclinic symmetry for fractured shales.

Figure 1: Marcellus shale, Pennsylvania, USA. Because of the fine-scale layering, this formation (where it does not include natural or artificial fractures) has polar anisotropic symmetry. This symmetry is also apparent on the grain scale, too small to be seen in this photograph.

Apparent isotropic parameters from log data

In this section, we assume that the underlying data come from sonic (V_P and V_S) and density (ρ) logs, from a wellbore which is vertical and normal to the bedding planes, thus parallel to the polar anisotropic symmetry axis. We also assume that the logs record these nominal parameters accurately, without any of the many issues that could interfere with this conclusion.

In this situation, these velocities are more properly called "vertical velocities" V_{P0} and V_{S0} . In this case, there is a common misconception that, for example, the apparent Young's modulus is the "vertical Young's modulus", i.e. measurable by conventional techniques, in the vertical direction, but calculated using the isotropic formulae:

$$\rho V_S^2 = \mu \qquad (1a)$$

$$\rho V_P^2 = M = K + \frac{4}{3}\mu \qquad (1b)$$

$$\lambda \equiv M - 2\mu \qquad (1c)$$

$$E \equiv \frac{\mu(3M - 4\mu)}{(M - \mu)} \qquad (1d)$$

$$\nu \equiv \frac{M - 2\mu}{2(M - \mu)} \qquad (1e)$$

To understand why this is not true, recall that Young's modulus is defined as the elastic parameter which governs the strain in the direction of the applied stress (in this case vertical), while the *stresses* in the orthogonal directions are zero. In the laboratory, this is the axial strain, in response to axial stress, on a long, thin cylinder with free sides.

By contrast, the parameter ***M***, related to the P-wave velocity via (1b), is the elastic parameter which governs the strain in the direction of the applied stress (in this case vertical), while the *strains* in the orthogonal directions are zero. In the laboratory, this is the axial strain, in response to axial stress, on a cylinder with confined sides.

Young's modulus does not appear in the expression for the elastic velocities (1ab) in a natural way, but it does appear in the elastic compliance tensor (*cf., e.g.* Thomsen, 2010), which may be displayed, for isotropic media, as the symmetric matrix (2), in which only the non-zero components are shown. The indices of this matrix refer to various combinations of directions; many of these components are identical since, in isotropy, many of these direction-combinations are equivalent.

$$\overline{S}^{iso} = \left| \begin{array}{cccccc} \frac{1}{E} & \frac{-\nu}{E} & \frac{-\nu}{E} & & & \\ & \frac{1}{E} & \frac{-\nu}{E} & & & \\ & & \frac{1}{E} & & & \\ & & & \frac{1}{\mu} & & \\ & & & & \frac{1}{\mu} & \\ & & & & & \frac{1}{\mu} \end{array} \right| \qquad (2)$$

For a polar anisotropic medium with symmetry axis vertical, the compliance tensor may be displayed as equation (3). Here one sees the horizontal and vertical Young's moduli $\boldsymbol{E_{11}}$ and $\boldsymbol{E_{33}}$; the 22 component is also named with $\boldsymbol{E_{11}}$, since the 1-direction and the 2-direction are equivalent, for this symmetry. One also sees directional shear moduli $\boldsymbol{\mu_{13}}$ and $\boldsymbol{\mu_{12}}$, and directional Poisson's ratios $\boldsymbol{\nu_{12}}$ and $\boldsymbol{\nu_{13}}$. There are only 5 independent parameters, since $\nu_{12} = E_{11} / 2\mu_{12} - 1$.

$$\overline{S}^{plr} = \left| \begin{array}{cccccc} \frac{1}{E_{11}} & \frac{-\nu_{12}}{E_{11}} & \frac{-\nu_{13}}{E_{33}} & & & \\ & \frac{1}{E_{11}} & \frac{-\nu_{13}}{E_{33}} & & & \\ & & \frac{1}{E_{33}} & & & \\ & & & \frac{1}{\mu_{13}} & & \\ & & & & \frac{1}{\mu_{13}} & \\ & & & & & \frac{1}{\mu_{12}} \end{array} \right| \qquad (3)$$

However, a description of the elastic *velocities* of polar anisotropic media requires the use of the elastic *stiffness* tensor, which is the mathematical *inverse* of the elastic compliance tensor (3). The stiffness tensor corresponding to (3) may be displayed as the matrix:

$$\tilde{C}^{plr} = \rho \left| \begin{array}{cccccc} V_{P90}^2 & \lambda_{12} / \rho & \lambda_{13} / \rho & & & \\ & V_{P90}^2 & \lambda_{13} / \rho & & & \\ & & V_{P0}^2 & & & \\ & & & V_{S0}^2 & & \\ & & & & V_{S0}^2 & \\ & & & & & V_{S9090}^2 \end{array} \right| \qquad (4)$$

In equation (4), the 33 component is labelled ρV_{P0}^2; this is density times the vertical P-velocity squared. By convention we use the subscript 0 (to indicate zero angle of incidence) instead of 3 (to indicate the 3-axis). Similarly, the 11 and 22 components, controlling the horizontal P-velocity, are labelled ρV_{P90}^2. The 44 and 55 components, controlling the vertical S-velocity, polarized horizontally, are labelled ρV_{S0}^2. And the 66 component, controlling the horizontal S-velocity, polarized horizontally, is labelled ρV_{S9090}^2.

There are two different Lame parameters, shown as the 12, 13, and 23 components. The 12 component is not independent of the others, but is given by the expression:

$$\lambda_{12} = \rho V_{P90}^2 - 2\rho V_{S9090}^2 \qquad (5)$$

So there are five independent stiffnesses; the form of the stiffness matrix (4) is very similar to that of compliance matrix (3).

It is conventional to express the anisotropy which is evident in equation (4) via three parameters: ***δ, ε,*** and ***γ*** , defined implicitly by:

$$V_{P90}^2 = V_{P0}^2(1+2\varepsilon) \qquad (6a)$$

$$V_{S9090}^2 = V_{S0}^2(1+2\gamma) \qquad (6b)$$

$$\lambda_{13} = \rho\left(V_{P0}^2 - 2V_{S0}^2 + V_{P0}^2\delta\right) \qquad (6c)$$

Then the five independent parameters maybe taken as V_{P0}, V_{S0} , ***δ, ε,*** and ***γ*** . If the anisotropy is weak, this choice of parameters leads to simple expressions for the elastic velocity, as functions of the polar angle (Thomsen, 1986). Hence, they are familiar to many exploration geophysicists, so it is useful to express the *compliance* anisotropy, which is evident in equation (3), in these same terms.

Conventional sonic logs only measure V_{P0} and V_{S0}; they do not measure ***δ, ε,*** or ***γ***. So, if we want to estimate Young's modulus from such data, one option is to ignore the anisotropy, and define apparent isotropic parameters (with subscript 0) using the vertical velocities V_{P0} and V_{S0} (*cf.* equation (1) :

$$\rho V_{S0}^2 \equiv \mu_0 \qquad (7a)$$

$$\rho V_{P0}^2 = M_0 = K_0 + \frac{4}{3}\mu_0 \qquad (7b)$$

$$\lambda_0 \equiv M_0 - 2\mu_0 \qquad (7c)$$

$$E_0 \equiv \frac{\mu_0(3M_0 - 4\mu_0)}{(M_0 - \mu_0)} \qquad (7d)$$

$$\nu_0 \equiv \frac{M_0 - 2\mu_0}{2(M_0 - \mu_0)} \qquad (7e)$$

So, now we can pose the question: what is the difference between the true vertical Young's modulus E_{33} (in equation (3)), and the apparent Young's modulus E_0, calculated from the vertical velocities using the isotropic equation (7d) ? Closed-form expressions exist (*e.g.* Nye, 1985) for the stiffness components, in terms of the compliance components, but they are non-linear and resistant to intuitive understanding.

However, in the common geophysical case that the anisotropy is weak, *i.e*. that the parameters, ***δ, ε,*** and ***γ*** are all <<1, the expressions simplify (Thomsen, 2010). In the notation used here, the apparent vertical Young's modulus E_0 is just the true vertical Young's modulus E_{33} with a correction depending linearly on the anisotropy:

$$E_0 = E_{33} - 4\nu_0 M_0\left[2\nu_0\varepsilon - \delta\right] + 8\nu_0^2\mu_0\gamma \qquad (8a)$$

Perhaps surprisingly, the difference depends upon all three anisotropy parameters. The correction cannot be measured using log data; its magnitude may be either positive or negative, and may be significant or not, depending on these .anisotropy parameters.

The apparent Young's modulus E_0 is also related to the true *horizontal* Young's modulus E_{11} by the expression (Thomsen, 2010):

$$E_0 = E_{11} - M_0(1-\nu_0)\left[\left(1-\nu_0\right)2\varepsilon - 2\nu_0\delta\right] - 8\nu_0(1-\nu_0)\mu_0\gamma \qquad (8b)$$

It is often not clear whether it is the vertical or horizontal Young's modulus which is implied by those who use isotropic parameters to describe anisotropic rocks; in either case an error is incurred (cf. equations (8ab)).

A similar analysis applies to the apparent Lame parameter λ_0 (7c). Two Lame-type parameters appear in equation (4): λ_{12} (5) and λ_{13} (6c). The relations between these and λ_0 are derived, from (5, 6), as:

$$\lambda_0 = \lambda_{12} - 2\rho V_{P0}^2\varepsilon + 4\rho V_{S0}^2\gamma \qquad (9a)$$

and, from (6,7), as

$$\lambda_0 = \lambda_{13} - \rho V_{P0}^2\delta \qquad (9b)$$

It is often not clear which anisotropic Lame parameter is implied by those who use isotropic parameters to describe anisotropic rocks; in either case an error is incurred (cf. equations (9ab)).

A similar analysis applies to the apparent Poisson ratio ν_0 (7e). Two Poisson ratios appear in equation (3): ν_{12} and ν_{13}. The relations between these and ν_0 are derived, from (3, 6), as:

$$\nu_0 = \nu_{12} - \frac{2M_0}{E_0}(1-\nu_0^2)\left[(1-\nu_0)\varepsilon - \nu_0\delta\right] + 2\frac{1-\nu_0^2+2\nu_0^3}{(1+\nu_0)}\gamma \qquad (10a)$$

and

$$\nu_0 = \nu_{13} + (1-\nu_0)\left[4\nu_0\varepsilon - \delta\right] - \frac{2\nu_0\left(1-3\nu_0\right)}{(1+\nu_0)}\gamma \qquad (10b)$$

It is often not clear which anisotropic Poisson's ratio is implied by those who use isotropic parameters to describe anisotropic rocks; in either case an error is incurred (cf. equations (10ab)).

Apparent isotropic parameters from surface seismic moveout

In this section, we assume that the underlying data come from seismic moveout (V_{PNMO} and V_{SNMO}), as determined for a particular depth or time interval in the subsurface. Two shear modes, SH (polarized horizontally) and SV (polarized perpendicular to SH), propagate, with different velocities, in polar anisotropic formations. The short-spread moveout velocities are related to the corresponding vertical velocities by (Thomsen, 1986):

$$V_{PNMO} = V_{P0}(1+\delta) \qquad (11a)$$

$$V_{SVNMO} = V_{S0}\left(1+\frac{V_{P0}^2}{V_{S0}^2}(\varepsilon-\delta)\right) \qquad (11b)$$

$$V_{SHNMO} = V_{S0}\left(1+\gamma\right) \qquad (11c)$$

where δ, ε, and γ are the same anisotropy parameters introduced in equation (6). The results of the previous section apply, with the additional corrections implied by equations (11abc).

Apparent isotropic parameters from surface seismic reflectivity

In this section, we assume that the underlying data come from reflection amplitudes. The plane-wave P-reflection coefficient for a horizontal planar interface between polar anisotropic formations, linearized for small elastic contrasts, is (Rueger, 1998):

$$\mathbf{R}_p(\theta_w) \cong \mathbf{R}_0 + \mathbf{R}_2 \sin^2\theta_w + \mathbf{R}_4 \sin^2\theta_w \tan^2\theta_w \qquad (12)$$

where θ_w is the angle between the vertical and the wavefront normal, and the coefficients are:

$$\mathbf{R}_0 = \frac{1}{2}\left[\frac{\Delta V_{P0}}{\bar{V}_{P0}} + \frac{\Delta\rho}{\bar{\rho}}\right] \qquad (13a)$$

$$\mathbf{R}_2 = \frac{1}{2}\left[\frac{\Delta V_{P0}}{\bar{V}_{P0}} - \left(\frac{2\bar{V}_{S0}}{\bar{V}_{P0}}\right)^2 \frac{\Delta\mu_0}{\bar{\mu}_0} + \Delta\delta\right] \qquad (13b)$$

$$\mathbf{R}_4 = \frac{1}{2}\left[\frac{\Delta V_{P0}}{\bar{V}_{P0}} + \Delta\varepsilon\right] \qquad (13c)$$

This differs from the isotropic AVO equation principally through the explicit inclusion, in equations (13bc), of the contrasts (across the reflector) in the anisotropy parameters, $\Delta\delta$ and $\Delta\varepsilon$, of the formations.

The remarkable thing about these equations is that, although the anisotropic contributions here are small (<<1), *all* of the other terms are *also* similarly small (this is a fundamental assumption of the linearization process). There is no leading term which is not small, as in equations (11). Hence, the neglect of the anisotropic terms may lead to large percentage errors, +/- 100% or more, even including reversal of algebraic sign.

Usually, this issue is addressed by normalizing the received seismic amplitudes in Common Depth Point reflection gathers, near a wellbore, to the reflection coefficient calculated from sonic and density logs in the well. Of course, this calculated reflection coefficient is necessarily isotropic, *e.g.* using equations (12, 13) with the anisotropy contributions assumed zero. This produces estimates of vertical elastic properties, reducing the problem (near the wellbore) to that of discussed above (equations (1-10)), except band-limited in resolution.

Additional difficulty arises if this same normalization is extended to other CDP gathers centered away from the wellbore. The problem is that the normalization is a *multiplicative* correction, whereas equation (13) shows that a proper correction (to find vertical elastic properties) is *additive*, not multiplicative. A positive multiplicative normalization cannot correct for a change of algebraic sign, whereas an additive correction can do this, on those occasions where it is required by the magnitude of the anisotropic corrections.

Furthermore, an adequate method of subsurface characterization should provide estimates of the missing anisotropy parameters, with the same spatial resolution. Currently, there is no accepted method for accomplishing this.

Conclusions

This work attempts to understand how apparent isotropic elastic parameters, derived from various types of data on anisotropic formations, are related to the true anisotropic elastic parameters of those formations. The relationship depends on the type of data that forms the basis for the apparent isotropic parameters, and the symmetry of the anisotropic formation. For clarity and specificity, the present analysis is restricted to the case of weak polar anisotropy; the present methods can be extended directly to lower symmetries.

In all cases, the apparent isotropic parameters differ from the true anisotropic parameters because of the anisotropy. These differences are expressed here in closed form, in terms of the anisotropic parameters δ, ε, and γ, which are familiar from the seismic context. The anisotropic corrections may lead to significant differences in conclusions, depending on the context.

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